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ON THE USEFULNESS OF MATHEMATICAL STUDIES.

The study which appears to be most likely to produce a habit of thinking, is the mathematics; on this account it is rather surprising that it does not constitute an essential part of the education of youth. Beginning from the simplest ideas, evidence of which is within the reach of the most obtuse, or youthful understanding, their progress is along a gentle declivity, never interrupted by precipices, chasms, or impending rocks. Starting from axioms, whose truth rests on sensible bases; from definitions clear, precise, and absolute; from postulata that cannot be objected to under any pretence whatever—they proceed from proposition to proposition in one close and regular concatenation. Every step is directed towards a distinct and single object, and that by the shortest and straightest road. Every one of those steps is a resting place, from which we can take a retrospective view of that which, by previous exertion, we had made our own; and, looking forward, new treasures are discovered, which stimulate as rewards to obliterate the remembrance of past labours, and brace up the faculties to fresh exertions. In the mathematics nothing is left to hypothesis, conjecture, or supposition; here no metaphysics bewilder, nor controversial bog obstructs the journey. The mind is never led astray from the direct path, by divergent ideas: it seizes

them as they suddenly and naturally arise from the subject under contemplation. But its powers are concentrated on a single one, of which it must be master before it can go on to another. We may compare *Synthesis* to the luxuriant foliage of the forest, the beautiful verdure of which commands our admiration; and *Analysis* to the no less attractive enamel of the meadow. Under the one we rest with delight; and cull with rapture the flowers which adorn the other. Superior to the logic of the schools, these sciences never show their power by giving to error the semblance of truth; they do not attempt to substitute specious and dazzling rhetorical figures to the stern syllogism of eternal truth, or to lull the judgment by a flow of select and high-sounding words, and still less, to captivate our suffrage by eloquence: but their language, however unadorned, always carries with it the most complete conviction, and scepticism vanishes before it. Far from fostering that bane to rationality, prejudice, they accustom the mind to admit of no position as certain, but upon the clearest and most evident demonstration; and to repel, with noble indignation, every attempt to impose upon it opinions resting merely on the name and fame of those who advance them. If we consider the universality, and the more or less direct connection which mathematics have with almost every branch of man's ingenuity and industry, our reverence for its principles will increase. Behold yon ship, examine her curvilinear form, the obliquity of her masts, the direction and position of her sails: mathematical theory has directed and sanctioned the whole. Examine that wind-mill, and every part of it will be found to be constructed upon mathematical principles. Open that watch, and you will not see in it the smallest cog which has not the shape prescribed by mathematics. To these sciences we owe the form of the lenses which empowered Herschel to penetrate into, and make discoveries in, the empyreal vault. To them Newton was indebted for assisting him in tearing asunder the veil, behind which nature had

deposited the tables of the laws which regulate the incessant and variable motion of the ponderous bodies that float in infinite space. To trace out all the various ramifications would be endless; I cannot conclude, however, without endeavouring to impress upon the minds of youth their vast importance in the scale of civilized society, and of assuring them, that, far from being either of a cloying or surfeiting nature, the allurements of mathematics increase in a progression whose last term is lost in infinity.

CHEVALIER D'ESTIMAVILLE, *Quebec.*

ELECTRICAL FACT.

Every thing in nature appears to be, in some measure, impregnated with that property, or perhaps substance, which the ancients called *anima mundi*, or elementary fire. Now, as all things have this fire in a greater or less proportion, only as they are in this or that place, where more or less is offered to be received by them, or as they are in their nature capable of receiving more of it than others are; and then, if we suppose the nature of the sensitive plant is to possess more of this fire than any other plant whatever; then it must, by its nature, when touched, impart a great deal of its fire or electricity into the thing by which it is touched; because the thing which touched it had less than was possessed by the plant: therefore, till the sensitive plant has had time to collect from the air a fresh supply of this fire, its leaves and branches hang in a languid state, from the loss it had sustained by its electricity having been discharged. To illustrate this hypothesis: if you set any small plant or tree in a pot, upon a cake of resin, and then electrify the tree, even though it be a willow, whose leaves are long and slender, yet it will be so acted upon, that its leaves will be raised up so as to create a considerable degree of surprise in the observer; and the moment you touch

only one of its leaves, the whole tree becomes languid, just like the sensitive plant, when touched in the manner above described.

E. R.

METHOD TO OBTAIN PURE SILVER.

To those who may want a small quantity of very pure silver, the following process is recommended; which is both easy and economical.

Dissolve half a dollar in diluted nitrous acid with heat; dilute the solution with about twice its weight of distilled or rain water, and drop in muriatic acid as long as any precipitation is visible; then put it upon a filter, and wash it with distilled or rain water, till the water comes off nearly tasteless: dry this muriat of silver, and triturate it with about four times its weight of dried carbonate, or sub-carbonate of soda. Put it into a covered crucible, that will hold near twice the quantity, and place it in a wind furnace, or a Blacksmith's forge, (a violent heat not being necessary, only, it must be applied quickly to prevent the muriat of silver from flying off,) in the space of ten minutes a lump of pure silver will be formed at the bottom of the crucible.

It is very evident that unless a pure silver be made use of a pure nitrat, muriat, or acetat cannot be expected, all of which are such useful and delicate tests for muriat of soda, and muriatic acid.

D. Dann.

ON GRAVITATION.

The different systems for explaining the cause of gravity, are those of Descartes, Newton, Bernouilli, Le Sage, and Boscovich. Respecting the system of Descartes, it is not necessary to enter into much detail; the vortices of that ingenious theorist have

long ceased to afford any satisfaction even to the most superficial reasoner. They are now known only in the history of opinions, and in that history will ever furnish a most instructive chapter.

The next system, for this purpose, was that of elastic ether, by Newton, in the queries at the end of his Optics, and proposed with such modesty and diffidence as entitles it to great indulgence. It should be remembered that it is the conjecture of the philosopher who had demonstrated the existence of the law of gravitation, concerning the mechanism by which this universal tendency is produced, and seems to be thrown out with a view of preventing those who followed him from thinking that it was sufficient to say that gravitation was an essential quality of matter, and that they had no occasion to trouble themselves about the cause of it. It was to serve as a stimulus to future inquiry, and as a caution against supposing that the Fabric of Physical Astronomy was complete. According to it, the mutual tendency of bodies towards eash other, arises from the action of a fluid highly elastic, diffused through all space, but more rare within bodies than without, and more rare at a smaller distance from them than at a greater, Bodies are propelled through this fluid from the denser to the rarer parts; that is, from the parts where the elasticity is greater, to those where it is less. Thus with respect to the earth, the elasticity of the circumfused ether being greater at a distance from that body than near it, other bodies would, by that greater elasticity, be urged to the quarter where the elasticity is less, that is, towards the earth. The same would hold of the sun and moon, and all the great bodies of the universe. This hypothesis, to which many objections may be made, appears to have been suggested to Newton by the phenomena of optics, which it is better calculated to solve than those of astronomy.

The system of John Bernouilli was proposed long after that of Newton, and intended by the author to unite the advantages of the Cartesian and Newtonian systems, without being subject

to the difficulties of either; but which, in the general opinion of philosophers, does exactly the reverse, uniting the difficulties of both, without the advantage of either. The truth is, Bernouilli, though one of the greatest mathematicians of the age, was far from deserving the same rank among philosophers. His theory of gravity is accordingly almost forgotten, and is at present little known, even to men of science.

Another system was proposed by Le Sage, in his "Essai de Chimie Méchanique," first published in 1758. This system is very little known; the following is a brief outline.

"Imagine that through all space numberless corpuscles, or atoms, almost infinitely small, are in perpetual motion: that every corpuscle has its determined direction, and moves for ever in a straight line, with a velocity far exceeding that of light. It is evident that the direction of the corpuscles may be so various, they may be themselves so small, and their velocities so great, that though they follow each other at vast distances, and leave space, in reality, almost empty, yet may they abound every where in such a manner, that in a portion of time, almost infinitely small, a great number of them may pass through every point of space whatsoever. On whatever point of space, therefore, our attention is fixed, we may consider it as a centre, to which the motions of an infinite number of atoms are referred, either diverging from it, or converging to it.

This constitution of what Le Sage calls the *gravivic fluid* being conceived, suppose a solid body to be plunged in it, of any figure, larger than one of the corpuscles, and in some degree, if not wholly, impervious to those corpuscles. This body will remain at rest, or at least without any progressive motion, the impulses from the corpuscles that strike against it, being equal in opposite directions. It may oscillate a little backwards and forwards, but will not be forced from its place. Now, let there be plunged into the gravivic fluid another body of any figure, and at any distance from the first; these two bodies will immediately begin to move towards one another: for the

one serving to protect the other from a certain quantity of the impulsion of the corpuscles, the current thus left without opposition necessarily produces the effect, and impells the bodies towards each other. Their motion towards one another will be continually accelerated ; and the force producing that acceleration will increase in proportion as one body stops more of the currents from falling on the other ; that is nearly as the squares of the distances diminish.

Again, if the solid particles of which the bodies are made be impenetrable to the gravivic fluid, but the bodies themselves on account of their porosity, permeable by them in a certain degree, the number of corpuscles that are stopped by each of the bodies will be, *cœteris paribus*, proportional to the number of solid particles, that is, to the quantities of matter in the bodies ; and hence, in general, the force urging the bodies towards one another will be directly as the masses and inversely as the squares of their distances. Thus by mechanical action, the Newtonian law of gravitation is explained in all its parts."

The principal objection against this theory is grounded on the vast expense of matter required to continue the existence of the gravivic fluid. No particle of that fluid returns to its place, or ever passes a second time through the same point of space. A constant supply, therefore, of new matter, or particles, is always necessary, as all those that are contained within the limits of the sensible universe at any instant, must be replaced long before they have entirely escaped from it, and gone forth for ever to traverse the deserts of uninhabited extension. The imagination is terrified at this constant exertion of what cannot be considered as less than creative power, employed in producing existences, that for a limited time are to be useful, and through all the rest of infinite duration are to be entirely useless. The difficulty is, indeed, great, and we are still brought within sight of the CAUSE to which all others are subordinate, the CAUSE which all rational systems acknowledge,

and can only differ about the point, where its immediate action begins, and beyond which secondary causes cannot be traced.

Boscovich's system does not, strictly speaking, assign the *cause* of gravity; it only generalizes the facts concerning the actions of bodies on one another, and reduces them to a single one.

P. Thompson.

USEFUL RECEIPTS.

To Preserve Iron from Rusting.

To any quantity of fat oil varnish, add four-fifths of rectified spirits of turpentine, apply this varnish lightly, but equally, with a sponge, and put the articles in a dry place, sheltered from dust.

To Clean Books.

A solution of the oxalic, citric, or tartareous acids, in water, will discharge *writing*, but not *printing* ink. Hence they may be employed in cleaning books, which are defaced by writing in the margin, &c. without injuring the text.

A Varnish for Transparencies.

Take equal parts of balsam of Canada, and spirits of turpentine, and dissolve with a gentle heat. No preparation previous to its being used is required; such parts as are to be quite transparent, must be done on both sides, but such as are to be less so, only on one side.

LITERARY NOTICES, &c.

We understand that Professor Day, of Yale College, has nearly completed his very useful Course of Mathematics, published in parts.

Professor Adrain, A. M. of Columbia College, New-York, has in the press a *third* edition of Dr. Hutton's Course of Mathematics.

Professor Coxe, M. D. of Philadelphia, has published No. 1 of a new periodical work, entitled, the "American Medical Recorder," for January, 1818, composed chiefly of selections from foreign journals.

Olinthus Gilbert Gregory, L. L. D. of the R. M. Academy, Woolwich, has lately published a small, but elegant little volume, on Trigonometry; which, we believe, has not yet been brought to this country.

Professor Leybourn, of the R. M. College, near Bagshot, still continues the second series of the Mathematical Repository, a number of which has lately appeared; this is the best work of the kind in Europe.

Dr. Thompson has re-written and published his System of Chemistry, in 4 vols. 8vo. and, without adopting the new Nomenclature of Davy, Berzelius, and some other modern reformers, has brought the state of chemical science down to the present time.

Orfila, so renowned for his Toxulogy, is now busily engaged in preparing a new system of chemistry.

Extract of a Letter from Dr. Bostock, dated London, November 2d, 1817.

Scarcely had I arrived here, when a new office was conferred upon me. Dr. Thompson, the editor of the "Annals of Philo-

sophy," has been elected Professor of Chemistry in the University of Glasgow; of course, was not able to continue the business of the editorship. This has been undertaken by Mr. Arthur Aikin and myself. Any communications on scientific topics we shall be glad to receive from our friends in America.

I have one or two queries to make, on subjects connected with natural history. During the last four or five years, a particular kind of disease has attacked the *plane trees*, in all parts of this island; so that, from being a tree which grew very luxuriantly with us, the whole race is nearly destroyed. As it is a native of your country, and was, indeed, imported from America, I wish to know whether any similar complaint has occurred to these trees in their native place. It may be necessary to state, that the tree to which I refer, is the *Platanus Occidentalis.*

A Biographical Sketch of the late Dr. Wistar, of Philadelphia.

We trust that the following extract from the Eulogy, delivered by Dr. Hosack, in the Hall of the College of Physicians and Surgeons, in New-York, January 26th, 1818, will be acceptable to our readers.

To record the talents and virtues of great men, is a delightful employment; and to hold them up as examples to the rising generation, is a duty which naturally devolves upon every journalist, who is anxious to preserve, in the memory of the living, a due regard for the excellencies of the illustrious characters who have adorned great stations—the worthies of every land. A small portion of each number will be appropriated to this useful purpose, and short biographical notices of eminent persons, recently deceased, will be gratefully received.

"Dr. Caspar Wistar was a native of Philadelphia, in which city he was born in the year 1760. His parents were of German

extraction, and belonged to the Society of Friends, of which they were highly respected members. Young Wistar received the elements of scholastic learning at the grammar school in Philadelphia, which was originally founded by William Penn. At that seminary he received an excellent English and Classical education, the institution at that time being under the direction of Mr. John Thompson, an eminent scholar, and a very able teacher of the Latin and Greek languages, and now a respectable merchant in that city. With this preparatory knowledge, he fixed upon the study of medicine, as the business of his future life, and became a pupil to the late Dr. John Redman, a very eminent practitioner of physic in Philadelphia. He also availed himself of the opportunity of attending the lectures of Drs. Morgan, Shippen, Rush, and Kuhn. Stimulated by the success and distinction which these eminent men had derived from a visit to Europe, he took his departure for Edinburgh, in the spring of 1784. Here he was distinguished for the same assiduity, correct moral deportment, and modest demeanour, as at home; and such was the impression made upon his friends at the university, that his name was ever afterwards mentioned in terms of the warmest regard and respect.

"In the year 1786. he returned to hisnative city, instructed in every branch of medicine, and the physical sciences with which it is connected. Soon after his return he was made professor of Chemistry and Physiology in the *College of Philadelphia*, and afterwards, on the consolidation of this College with the *University of Pennsylvania*, he was associated with Dr. W. Shippen, as adjunct professor of Anatomy and Surgery in the University of Pennsylvania. Dr. Wistar was too much engaged in the practical duties of his profession to enjoy sufficient leisure for extensive literary undertakings. The writings, however, which he has left, cause us to regret that they were not more numerous. The *System of Anatomy* which he published, as a text book for his class, is well known to you; he also enriched

the Transactions of the American Philosophical Society with several interesting memoirs, and, in the volume which will shortly appear, he drew up an account of the life and labours of his late colleague, Dr. W. Shippen. As a literary character he was held in the highest estimation, and his house was the weekly resort of the Literati of Philadelphia ; and at his hospitable board, the learned Stranger, of every tongue and nation, found a cordial welcome. In 1815, he was made a member of the Literary and Philosophical Society of New-York, and when the Presidency of the American Philosophical Society for promoting Useful Knowledge, was vacated in 1816, Dr. Wistar was unanimously chosen to fill that honourable station : honorable, as having been previously filled by a Franklin, a Rittenhouse, and a Jefferson."

DEATH OF DE LUC, THE GEOLOGIST.

Died, on the 7th day of November, 1817, at his house in Windsor, England, after a painful and lingering illness, John Andrew De Luc, F. R. S. aged 90. He was author of several works of very great celebrity, among the most important of which were, his " Lettres sur l'Histoire de la Terre et de l'Homme," 5 vols. 8vo. " Recherches sur les Modifications de l'Atmosphere," 3 vols. 8vo. " Precis sur la Philosophie de la Bacon," 2 vols. " An Elementary Treatise on Geology," 1809. " Geological Travels in the North of Europe, and in England," 3 vols. 8vo. " Geological Travels in some parts of France," 1803.

MATHEMATICAL CORRESPONDENCE.

Solutions to some Select Algebraic Problems.

1. Given $x^2+y^2=a$, and $xy=b$, to find x and y.

To the 1. *equ.* add twice the second, and we have $a^2+2xy+y^2=a+2b$, the sqr. rooot of which is $x+y=\sqrt{(a+2b)}$.

Also from the 1. equ. take twice the 2. and we shall have $x^2-2xy+y^2=a-2b$, the root of which gives $x-y=\sqrt{(a-2b)}$. We have now given the sum and differences of two quantities, the solution of which is well known.

2. Given $x+y=a$, and $x^2+y^2=b$, to find x and y.

From twice the second subtract the square of the first, that is

$$\begin{array}{rl} \text{from } 2x^2 & +2y^2=2b \\ \text{take } x^2+2xy+ & y^2=a^2 \\ \text{and we have } x^2-2xy+y^2 & =2b-a^2 \end{array}$$

the root of which gives $x-y=\sqrt{(2b-a^2)}$ and we have the sum and difference of two quantities, as before.

Or thus.

Put half the sum of the numbers $=s$, and half their difference $=x$; then $a+x$ will be the greater, and $a-x$ the less number, and the sum of their squares, that is

$$\begin{array}{rl} \text{to } & s^2+2sx+x^2 \\ \text{add } & s^2-2sx+x^2 \end{array}$$

and their sum, $2s^2+2x^2=b$, from whence, by transposition &c. $x=\frac{1}{2}\sqrt{(b-2s^2)}$

3. Given $x+y=a$, and $x^2-y^2=b$, to find x and y.

Put half the sum, or half $a=s$, $x=$ half the difference then,

$s+x=$ the greater, and $s-x=$ the less number, and the difference of their squares,

that is, from $s^2+2sx+x^2$

take $s^2-2sx+x^2$

and we have $4sx=b$, whence $x=\frac{b}{4s}$

4. Given $x^2-y^2=a$, and $xy=b$, to find x and y.

To the square of the 1. equ. add 4 times the square of the 2,

that is, to $x^4-2x^2y^2+y^4=a^2$

add $4x^2y^2 \quad =4b^2$

and we have $x^4+2x^2y^2+y^4=a^2+4b^2$

the root of which, or $x^2+y^2=\sqrt{(a^2+4b^2)}$; to this add the 1. equ. or, $x^2-y^2=a$; and we have $2x^2=a+\sqrt{(a^2+4b^2)}$, and $x=\frac{1}{2}\sqrt{(a+\sqrt{(a^2+4b)})}$. In the same manner y is found by subtraction.

5. Given $x^2+xy=a$, and $xy+y^2=b$, to find x and y.

Add the two equa. together, and we have $x^2+2xy+y^2=a+b$, the root gives $x+y=\sqrt{(a+b)}$.

Hence $(x+y).\ x=x.\sqrt{(a+b)}=a$

and $(x+y).\ y=y.\sqrt{(a+b)}=b$

therefore $x=\frac{a}{\sqrt{(a+b)}}$, and $y=\frac{b}{\sqrt{(a+b)}}$

6. Given $xy=x^2-y^2=x^3+y^3$, to find x and y.

Put $y=zx$, then, $zx^2=x^2-z^2x^2$

and $zx^2=x^3+z^3x^3$

Divide the 1. equ. by x^2 and we have $z=1-z^2$, or

$z^2+z=1$, a quadratic, from whence $z=-\frac{1}{2}+\frac{1}{2}\sqrt{5}$, which call c. Divide the 2. equ. by x^2, and we have $z=x+z^3x$, or

$c=x+c^3x$; and $x=\frac{c}{1+c^3}=\frac{1}{2}$. Now $y=zx=cz=\frac{1}{2}c-\frac{1}{4}+\frac{1}{4}\sqrt{5}$

7. Given $\frac{1}{x}+\frac{1}{y}=a$, and $xy=b$, to find x and y.

The sqr. of the 1. equ. is $\frac{1}{x^2}+\frac{2}{xy}+\frac{1}{y^2}=a^2$

and 4 divided by the 2*d*. is $\frac{4}{xy}=\frac{4}{b}$

their difference gives $\frac{1}{x^2}-\frac{2}{xy}+\frac{1}{y^2}=a^2-\frac{4}{b}$

the root of which is $\frac{1}{x}-\frac{1}{y}=\sqrt{(a^2-\frac{4}{b})}$

but $\frac{1}{x}+\frac{1}{y}=a$,

therefore, $\frac{2}{x}=a+\sqrt{(a^2-\frac{4}{b})}$

and $x=\frac{2}{a+\sqrt{(a^2-\frac{4}{b})}}$

In the some manner y is found by subtraction.

8. Given $x^2+y^2=a$, and $(x-y)^2+b(x+y)=c$, to find x and y. Put $x-y=u$, and $x+y=z$; then $u^2+z^2=2a$, and $u^2+bz=c$; their difference gives $z^2-bz=2a-c$, a quadratic whence z is readily found, and then all the rest.

The above solutions are very elegant, and are given as patterns for students to imitate; several other questions may be solved by a similar process, which the ingenious in these matters will easily discover, without further assistance

To determine the Differentials, or Fluxions, of Sines, Cosines &c.

The Arithmetic of Sines, or Analytical Trigonometry, has of

late been much cultivated in France; it is exceedingly useful in Physical Astronomy, and many branches of Natural Philosophy. The results obtained by this Calculus, are frequently more easily come at, than by other methods; and they are often vastly neater, and possess more generality, and of course are more elegant, than those obtained by other methods. Instances of what has been affirmed will be given in the course of this work, and, as an incitement to Students, we shall proceed to show how to find the Differentials, or, which is the same, the Fluxions, of Sines, &c. As there is no difference between the two methods except in the notation, the Differential Calculus will be preferred, because, in the present case, it is more commodious. Here dx is the same as $\dot{x}$, and dy the same as $\dot{y}$, &c.

1. Let $\sin. x = y$, then, we shall have $y + dy = \sin. (x + dx) = \sin. x \cos. dx + \sin. dx \cos. x$. Now dx being an indefinitely small arc, we shall have $\cos. dx = 1$, and $\sin. dx = dx$; therefore, $y + dy = \sin. x + dx \cos. x$, or $dy = d(\sin. x) = dx \cos. x$; that is, $=$ the differential of the arc, multiplied by its cosine. Or, the *Fluxion* of the sine of any arc is $=$ the Fluxion of the arc, multiplied by its cosine.

2. Since $dx \cos. x = d \sin. x$, if we make $x = 90^\circ - y$, we shall have $dx = -dy$, and $d \cos. y = -dy \sin y$.

Or thus, $\sin^2 x + \cos^2 x = 1$, therefore, $\sin x d \sin x + \cos x d \cos x = 0$, and $d \cos x = -\frac{\sin x}{\cos x} \times d \sin x = -\frac{\sin x}{\cos x} \times dx \cos x = -dx \sin x$, as before.

Again, $d \cos x = \cos (x + dx) - \cos x = \cos x \cos dx - \sin dx \sin x - \cos x = -dx \sin x$, as before.

3. Let $\tan z = \frac{\sin x}{\cos x}$, then we have $dz = \frac{\cos x d \sin x - d \cos x \sin x}{\cos^2 x} = \frac{dx \cos^2 x + dx \sin^2 x}{\cos^2 x} = \frac{dx}{\cos^2 x}$, (because $\cos^2 x + \sin^2 x = 1$.) $= d \tan x$.

4. Let $x=90^{\circ}-y$, then $d\ cot\ y=\frac{-dy}{sin\ y}$

Or, since $cot.\ z=\frac{cos\ x}{sin\ x}$, we have $\frac{d\ cos\ x\ sin\ x-d\ sin\ x\ cos\ x}{sin^2\ x}$

$=\frac{-dx\ sin^2\ x-dx\ cos^2\ x}{sin^2\ x}=\frac{-dx}{sin^2 x}$.

5. To find the differential of $sec\ x$; we have $d\ sec\ x=d\frac{1}{cos\ x}$

$=\frac{-d\ cos\ x}{cos^2\ x}=\frac{dx\ sin\ x}{cos^2\ x}=\frac{dx\ tan\ x}{cos\ x}$.

6. Also, $d\ cosec\ x=d\left(\frac{1}{sin\ x}\right)=\frac{-d\ sin\ x}{sin^2\ x}=\frac{-dx\ cos\ x}{sin^2\ x}=\frac{-d\cot x}{sin\ x}$

These are the principle rules, which we shall endeavour to elucidate by some examples.

Ex. 1. First, $d\ (sin\ 3\ x)=3\ dx\ cos\ 3\ x$; also $d\ (sin\ mx)=mdx\ cos\ mx$.

Ex. 2. Also, $d\ (cos\ 2x)=-2dx\ sin\ 2x$; also, $d\ (cos\ mx)=-mdx\ sin\ mx$.

Ex. 3. Moreover, $d\ (sin\ x)^m=m\ (sin\ x)^{m-1}\times sin\ dx=mdx\ cos\ x\times(sin\ x)^{m-1}=m\ sin\ x^m\ dx\ cot\ x$.

Ex. 4. Again, $d\ (sin\ x\ cos\ x)=cos\ x\times d\ sin\ x+sin\ x\times d\ cos\ x=dx\ cos^2\ x-dx\ sin\ x=dx\ cos\ 2x$.

Ex. 5. The $dd\ (sin\ x)=ddx\ cos\ x-dx^2\ sin\ x$.

Ex. 6. The $dd\ (cos\ x)=d(-dx\ sin\ x)=-dd\ sin\ x-dx^2\ cos\ x$

Ex. 7. The $d\sqrt{\left(\frac{1+cos\ x}{2}\right)}=\frac{1}{\sqrt{2}}d\ (1+cos\ x)^{\frac{1}{2}}=\frac{1}{2\sqrt{2}}(1+$

$cos\ x)^{-\frac{1}{2}}\times-dx\ sin\ x=\frac{-dx\ sin\ x}{2\sqrt{(2.(1+cos\ x)}}=\frac{-dx\ sin\ x}{4\ cos\ \frac{1}{2}\ x}=\frac{-dx}{2}\ sin$

$\frac{1}{2}\ x$; because $\sqrt{\left(\frac{1+cos\ x}{2}\right)}=cos\frac{1}{2}x$.

Ex. 8. The $d\,(\cos lx) = -dlx \sin lx = -\frac{dx}{x} \sin lx$.

Ex. 9. The $d\,(x \sin x) = dx \sin x + x\,dx \cos x$.

Ex. 10. If x be any arc, its differential $dx = \frac{d \sin x}{\cos x} = \frac{-d \cos x}{\sin x}$

$$= \cos^2 x\, d \tan x = \frac{d \tan x}{\sec^2 x} = \frac{d \tan x}{1 + \tan^2 x} = -d \cot x \sin^2 x = \frac{-d \cot x}{\operatorname{cosec}^2 x}$$

$$= \frac{-d \cot x}{1 + \cot^2 x}.$$

These examples will be sufficient to explain how the differential calculus may be applied to trigonometrical formulæ.

On the rectification of the Circle.

This is a subject to which mathematicians have, in all ages, paid considerable attention. To determine the exact ratio which exists between the diameter of a circle and its circumference, is necessary in order to determine its quadrature, but this still remains to be effected; and no reasonable hopes are now entertained of the possibility of ever accomplishing this great desideratum. Several approximations have at different times been invented for this purpose; the following converges very fast, it was discovered by Euler, and consists in finding the circumference of a circle from the tangent of 45 degrees.

Decompose the arc of 45° into two others, in this manner. We know that $\tan (a+b) = 1 = \frac{\tan a + \tan b}{1 - \tan a \tan b}$. Now if $a = \tan. 45° = 1$, then $1 - \tan b = \frac{\tan a - \tan b}{1 + \tan a \tan b}$, is the tangent of the difference of two arcs, that is (because $a = 1$) $= \frac{1 - \tan b}{1 + \tan b}$. Let $x = \tan b$, then our last expression becomes $\frac{1-x}{1+x}$, which put $= \frac{1}{2}$;

that is, $\frac{1-x}{1+x}=\frac{1}{2}$, and by reduction $2-2x=1+x$, or $3x=1$, whence $x=\frac{1}{3}$; and these two, namely $\frac{1}{2}$ and $\frac{1}{3}$, are the only two tangents whose sum is $=1$, according to the above formula. Therefore, substituting them in the series for the tangent of an arc, namely, $a=t-\frac{t^3}{3}+\frac{t^5}{5}-\frac{t^7}{7}+$ &c. we shall have the length of one fourth of the circumference of the circle; that is,

$$\frac{c}{4}=\begin{cases}\frac{1}{2}-\frac{1}{3}.\frac{1}{2^3}+\frac{1}{5}.\frac{1}{2^5}-\frac{1}{7}.\frac{1}{2^7}+\text{ \&c.}\\ +\frac{1}{3}-\frac{1}{3}.\frac{1}{3^3}+\frac{1}{5}.\frac{1}{3^5}-\frac{1}{7}.\frac{1}{3^7}+\text{ \&c.}\end{cases}$$

$=,785398163397$ &c.

MATHEMATICAL QUESTIONS,

TO BE ANSWERED IN NO. 2.

Qu. 1. By *A. B.*

Prove that $a^0=1$.

Qu. 2. By *Mr. Rt. Maar, New-York.*

Prove that Log. $3a^2+$Log. a^4+5 Log $3=$Log. $(3a)^6$.

Qu. 3. By *Mr. Hickman, Boston.*

Given $\frac{1}{x}+\frac{1}{y}=b$, and $x^2y+xy^2=a$, to find x and y.

Qu. 4. By *Mr. O'Carey.*

Given the sum of four numbers $=24$, the sum of their squares

=178, the sum of their cubes=1476, and the sum of their fourth powers=13042, to determine the numbers.

Qu. 5. By the *Editor.*

Given the length of a slender cylindrical wire=l inches, which bent at its middle point, in a given angle, forms the angular pendulum, vibrating in its own plane; supposing it to be suspended at the angular point, required the distance of the center of oscillation from the point of suspension, without having recourse to the center of gyration.

MATHEMATICAL QUESTIONS,

TO BE ANTWERED IN NO. 3.

Qu. 6. By *Mr. J. Cook.*

From a basket of apples A took 5, B. 7, and C. 10, A afterwards took half of what were left, and the remainder being [redacted] divided between B and C, they had all an equal number, how many had each?

Qu. 7. By *M. F. New-York.*

Given $x+y=110$, and Log. $x+$Log. $y=3$, to find x and y.

Qu. 8. By *Mr. M. O'Conner*, (Teacher of the Roman Catholic Lancastrian School, Barclay St.) *New-York.*

A Ship sailing from New-York Light-house, in lat. 40°. 28'. N. and lon. 74°. 7'. W. on a course between N. and E. found that her diff. of lat. and departure exceeded her distance by 229,1796 miles, and her diff. of lat. had to her departure the ratio of y to x in these equations.

$$y^3+yx^3=10,$$

$$\text{and } x^3+xy^3+x=10,$$

Required the investigation of the said ratio, and the lat. and lon. of the Ship.

Qu. 9. By *Mr. Rt. Maar.*

If the distance between the earth and sun be 95000000 of miles, and the earth's radius 3960 miles; required the sun's paralax.

Qu. 10. By *Analyticus.*

Given the three sides of the base of a triangular pyramid, and the three angles at the vertex, to find the solidity.

MATHEMATICAL QUESTIONS,

TO BE ANSWERED IN NO. 4.

Qu. 11. By *M. F. New-York.*

A. owed B. 1000 dollars, for which he was to allow 7 per cent in one year. A who was in a profitable business, proposed partnership upon condition that B. should allow him 15 per cent of his share of profits for managing &c. In 6 years they cleared 5000 dollars. How did their accounts then stand.

Qu. 12. By *Rt. Maar.*

Two men, A and B want to go to a place 10 miles distant, and having but one horse, agree to *ride* and *tie*; A is to ride the first mile and then walk forward, B is to walk the first mile and ride the second, and so on. Now if the horse can travel at the rate of 8 miles an hour, and each man at the rate of 4, how long will they be in performing the journey.

Qu. 13. By *M. O'Shannessy*, teacher of mathematics, Albany.

A certain youth was asked his age,
By one who seemed to be a Sage;

To whom the youth made this reply,
Sir, if you wish your skill to try,
Eight times my age increased by four,
A perfect square, nor less nor more;
Its tripled square plus nine must be
Another square, as you will see.
He tried but sure it posed him quite,
His answer being far from right.
You skill'd in Science I implore
This mystic number to explore.

Qu. 14. By *Mr. O'Conner, New-York.*

If the sines of two contiguous arcs of a circle be drawn from their point of division, and the points in which their limiting diameters are intersected by these sines, be connected by a straight line, this line will be equal to the sine of the sum of the said arcs; required the demonstration.

Prize. Qu. 15. By the *Editor.*

Let the weight of a Fly wheel=40, the distance of the center of gyration from the axis of motion=10, and the weight to move it equal 10; required the distance from the axis at which the weight must act, so as to produce the greatest number of revolutions in a given time.

The best solution to this question, will entitle the person who sends it to a number of the Journal for 12 months.